FIELD PROFILE IN A SINGLE-MODE CURVED DIELECTRIC WAVEGUIDE

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ABSTRACT

An approximate and simple method for predicting the field profile in a curved dielectric waveguide is described. It is shown that the field can be approximated inside the dielectric guide by the Airy function of the first kind and that the field decay coefficient assumes an integral form outside the guide. Experimental verificiation of the theoretical results is included.

Introduction

Recently, dielectric waveguides have seen increasing use as transmission lines for millimeter-wave integrated circuits. These lines are often curved to satisfy the small-size requirements of the system packaging. Also, many of the dielectric devices, such as ring resonators and couplers, are comprised of curved waveguides. Therefore, it is necessary to have a good understanding of the behavior of the field in the neighborhood of the bend in order to accurately predict the coupling and radiation characteristics. General analytical approaches[1,2] are currently available for analyzing curved waveguides. However, these approaches are typically rather involved in the mathematical sense and it is often useful to seek simple solutions which represent the field behavior in a curved dielectric waveguide in an approximate but accurate manner. The construction of this approximate solution proceeds as follows. We represent the fields in the inner region of the dielectric waveguide bend in terms of the solutions of the wave equation in an inhomogeneous linear medium.[3] In the outer region of the waveguide, the decay coefficient is modified from that of the straight guide because of the curvature. The validity of these approximate expressions is verified by experiments carried out at X-band.

Propagation Characteristics of a Curved Dielectric Waveguide

Consider a curved dielectric waveguide with a homogeneous refractive index n_1 surrounded by air, as shown in Figure 1. The bending of the guide is likely to lead to a distortion of the phase fronts of the wave being guided along the curved guide. The radial dependent of the guided wavelength in the curved structure can be approximated by[4]

$$\lambda(r) = \lambda_g \frac{r}{R} \qquad (1)$$

$$r = R + x \qquad (2)$$

where λ_g is the wavelength in the straight guide, and R is the mean radius of the curved guide.

Next, we transform the homogeneous curved guide into an inhomogeneous straight one via conformal mapping into the (w,z) plane as follows

$$w = R\ln\left(\frac{r}{R}\right) \qquad z = R \qquad (3)$$

w and z are the transverse and longitudinal coordinates of the linear structure, respectively.

In the transformed (w,z) plane, the longitudinal wave number is constant throughout the cross-section of the straight dielectric guide,[5] whereas in the original curved structure, the propagation constant varies according to Equation (1). In the transformed plane, the equivalent refractive index n(w) can be expressed[3] as

$$n(w) = n_1 e^{\frac{w}{R}} \simeq n_1 \left(1 + \frac{w}{R}\right) \qquad (4)$$

In view of this, the refractive indices at the inner and outer edges of the curved waveguide become $n_1(1 - \frac{a}{2R})$ and $n_1(1 + \frac{a}{2R})$, respectively, in the straight guide. With the increase of the refrative index as a function of r, the maximum in the energy distribution shifts toward the outer edge of the curved guide (see Figure 2). As expected, the degree of shift depends on the bending radius R. The wave equation for the electric field E in the dielectric waveguide becomes

$$\Delta E(w,y,z) + n^2(w)k_0^2E(w,y,z) = 0 \qquad (5)$$

where k_0 is the free-space wave number, and w(x) is the transverse coordinate in the (w,z) plane.

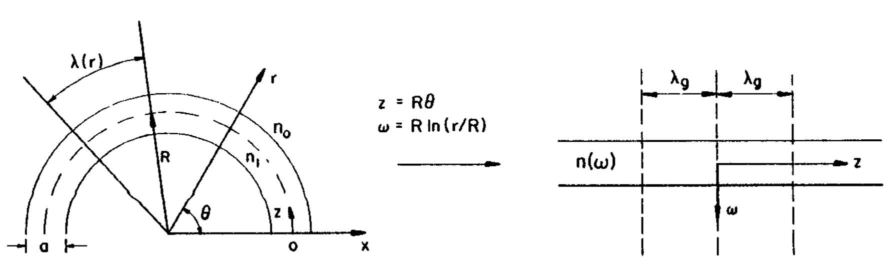

Figure 1. Curved dielectric waveguide and its transformation.

For relatively large radii, the w-variation component of the field in the curved guide can be expressed as:

$$\frac{d^2}{dw^2}\psi(w) + (\varepsilon_1 k_0^2 - k_y^2 - k_g^2 + \frac{2\varepsilon_1 k_0^2}{R} w)\,\psi(w) = 0 \quad (6)$$

where k_y and k_g are the transverse and axial propagation constants of the straight guide, and $\varepsilon_1 = n_1^2$.

Introducing a new variable given by

$$\zeta = (k_g^2 - \varepsilon_{re} k_0^2)(\frac{2\varepsilon_1 k_0^2}{R})^{-2/3} - (\frac{2\varepsilon_1 k_0^2}{R})^{1/3} w \quad (7)$$

the wave equation (6) may be rewritten as

$$\psi''(\zeta) - \zeta\psi(\zeta) = 0 \quad (8)$$

The linearly independent solutions for this equation are given by

$$Ai(\zeta),\ Bi(\zeta)$$

and $\quad (9)$

$$Ai(\zeta),\ Ai(\zeta e^{\pm 2\pi j/3})$$

The choice for the combination of these solutions for representing the field in a curved dielectric waveguide is not obvious. Given that the field maximum in a curved dielectric waveguide always shifts toward the outer edge and knowing the behavior of Airy functions[6], the solution can be approximately represented as

$$\psi(\zeta) \doteq C Ai(\zeta) \quad (10)$$

where C is an arbitrary normalizing constant, and ζ is related to the original transverse parameter x by Equations (3) and (7).

Using this expression, the field amplitude in the curved dielectric waveguide was plotted as a function of the transverse position x in the guide and compared with the experimental data as shown in Figure 3. It should be pointed out that the straight guide was designed such that it would propagate only the fundamental mode. Of course, a certain amount of mode conversion always takes place at the bend. However, the experimental results did not show the presence of these higher-order modes, at least not to a great extent.

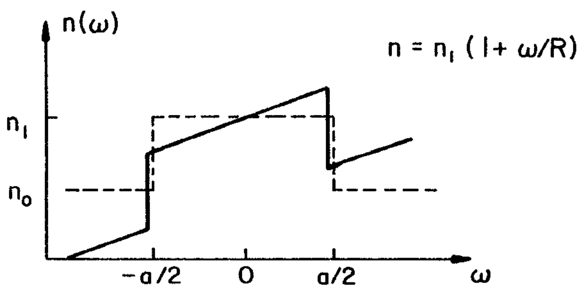

Figure 2. Transverse index profile in the (w,z) plane.

Outer-Region Fields in a Curved Dielectric Waveguide

From Equation (1), the axial wave number in a curved waveguide as a function of r is given by

$$k_z(r) = k_g(\frac{R}{r}) \quad (11)$$

Using the generalized effective dielectric constant method, the dispersion relation for the field decay coefficient outside the curved guide is given by

$$\xi^2(r) = k_z^2(r) - k_0^2 = k_g^2(\frac{R}{r})^2 - k_0^2 \quad (12)$$

This equation suggests that the field decay coefficient in a curved dielectric guide is no longer a constant but varies as a function of the distance away from the guide. On the outer side of the bend, ξ is real and positive in the region close to the dielectric guide, which implies that the field is still guided. At distances sufficiently far from the guide, the field decay coefficient becomes imaginary and this corresponds to a traveling wave in the radial direction. The critical distance at which this transform occurs is x_{cr}. On the outer side of the guide, in the region where is real, the radial variation of the field can be expressed as

$$\psi_{x_{cr}^-}(x) \sim \exp\left\{-\int_{a/2}^{x}(k_g^2(\frac{R}{R+\tau})^2 - k_0^2)^{1/2}\, d\tau\right\} \quad (13)$$

where x is the distance from the mean radius. The minus sign on x_{cr} indicates that the field is only valid in the region where $x < x_{cr}$.

Far from the guide, the field is given by

$$\psi_{x_{cr}^+}(x) = \psi_{x_{cr}^-}(x_{cr})\, H_\nu^{(2)}(k_0(R+x)) \quad (14)$$

where $\quad \nu = k_g R \quad (15)$

The choice of the Hankel function of the second kind is dictated by the requirement that the field must become an outward traveling wave for $x >> x_{cr}$.[4] The critical distance x_{cr} is defined as the distance x where

$$\psi'_{x_{cr}^-}(x) = \psi'_{x_{cr}^+}(x) \quad (16)$$

For sufficiently large R,

$$x_{cr} \doteq R(\frac{k_g}{k_0} - 1) \quad (17)$$

The field amplitudes derived from Equations (13) and (14) were plotted and compared with the experimental results, as shown in Figure 4.

On the inner side of the bend, the field decay coefficient is always real indicating that the field must decay in the direction of decreasing r. With ξ given in (12), the field profile in the inner side of the bend becomes

$$\psi_I(x) \sim \exp\left\{\int_{-\frac{a}{2}}^{x}(k_g^2(\frac{R}{R+\tau})^2 - k_0^2)^{1/2}\, d\tau\right\} \quad (18)$$

where x is negative in the region $r < R$. The experimental and theoretical results for the field amplitude in the inner side of the curved dielectric waveguide are plotted in Figure 5.

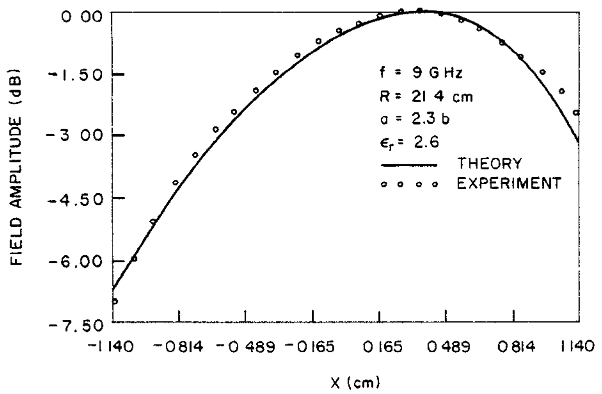

Figure 3. Field amplitude as a function of the transverse variable x in the guide.

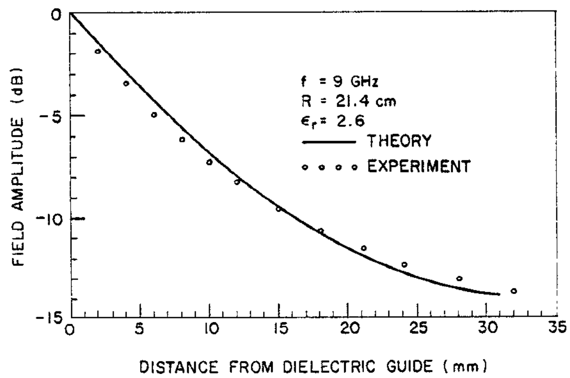

Figure 4. Field amplitude vs. distance from the guide on the outer side of a curved dielectric waveguide.

Conclusions

In this paper we have presented an approximate analytical method for predicting the field profile in a curved dielectric waveguide. Inside the dielectric waveguide, the field can be expressed as an Airy function. On the outer side of the guide, the field decay coefficient, which assumes an integral form near the guide, is expressed in terms of the Hankel function of the second kind at distances far away from the guide. On the inner side of the guide, the field decay coefficient can also be expressed in terms of an integral. Experimental verification of the field behavior predicted by the approximate theoretical formulas is included.

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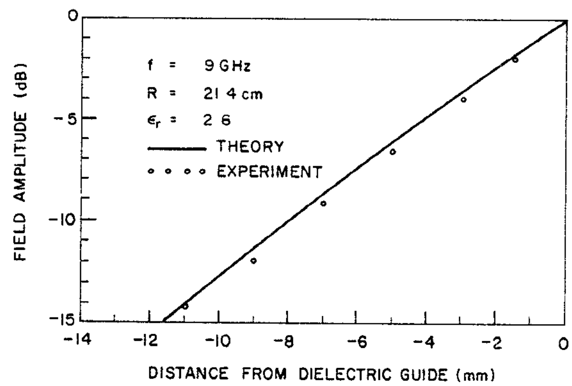

Figure 5. Field amplitude vs. distance from the guide on the inner side of a curved dielectric waveguide.